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Generalized Unsteady Solution of Isentropic and Isothermal Pressurization Process

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Abstract

Generalized numerical solutions of dimensionless pressure, mass and mass flowrate history of isentropic and isothermal pressurization of a rigid chamber have been developed from the results of parametrc studies. The physical parameters that can influence the solution are grouped to form a set of dimensionless quantities. The parametric studies are done by a computer program that models the physical processes associated with unsteady thermo-fluid dynamics. The computer program embodies a numerical scheme that has been developed to model the pressurization process. The numerical scheme solves for mass and energy conservation equations in conjunction with the equation of state by an implicit successive substitution procedure. The generalized solutions have been presented in the form of chart and tables for determining the pressure, mass and mass flowrate history of any given pressurization process.

1.0 Introduction

The pressurization of a chamber has a prominent place in the design of thermal systems in Aerospace Engineering. For example, this occurs in a solid rocket motor during ignition, in venting of a pressurized module in cryogenic fluid management. The behaviour of such systems during the transient operation is of paramount importance to address many safety issues related to design. The analysis of such systems involves time-dependant fluid flow and energy transfer. Unsteady forms of mass and energy conservation equations are required to be solved in addition to the flowrate equation and equation of state.

Moody[1] obtained an analytical solution of blowdown in a pressure vessel by assuming a constant pressure during the entire operation. By means of this assumption he was able to decouple the mass conservation equation from the flow rate equation. With the help of isentropic relationships he derived a first order differential equation of pressure. The differential equation was analytically solved as an initial value problem to predict the pressure history. In the aforementioned analysis the mass conservation equation was decoupled from the flowrate equation . The assumption of constant pressure resulted in a constant flowrate. During blowdown of a pressure vessel the variation of flowrate may not be that significant. As a result, Moody's predictions compared well with experimental data. However the variation of flowrate during pressurization was large enough so that decoupling of pressure could lead to an inaccurate result. In absence of any analytical method to solve for the system

of coupled nonlinear equations, the only recourse is to obtain a numerical solution of the governing equations. The numerical analysis of unsteady fluid flow and energy transfer problems can be classified in two categories, namely explicit and implicit methods. In an explicit method the mass and energy fluxes are calculated from the variable values prevailing at the previous time step. The implicit method, on the other hand, uses the varaible values at the current time step to calculate the fluxes. Since the values at the current time step are not known apriori the implicit method is iterative. The explicit method is simple to use. The disadvantage of the explicit method is that it is restricted to use only a small time step during the numerical integration process. The upper limit of time step size is not restricted by physical criterion, but must satisfy a numerical stability criterion. The time step size is less restricted in an implicit method, in general, due to the broader stability limits associated with this method. The timestep can be selected to satsfy the accuracy of predicted physical parameters. Due to the iterative nature of an implicit method it is more complex to use than an explicit method.

The present paper describes an implicit numerical method to predict the transient process during pressurization of a chamber using a control volume approach. The pressurization process has been studied by considering the filling up of an evacuated tank as shown in Figure 1. The time-dependent pressure, temperature and mass flowrate are calculated for a given supply pressure and temperature and the results of the numerical calculation have been presented in a generalized form. It is possible to determine pressure, temperature and mass flowrate history from the generalized plot. The duration of the transient process can also be calculated from a characteristic dimensionless number.

2.0 Computational Model

A control volume based procedure[2] is developed to solve the transient mass and energy conservation equations in conjunction with isentropic flowrate equation and equation of state. The governing equations for the system shown in figure 1 are now described.

2.1 Mass Conservation Equation:

The mass conservation equation can be written as

$$m_{t+\Delta t} = m_t + \dot{m}_{t+\Delta t}\,\Delta t \tag{1}$$

The term on the left hand side represents the mass of air in the chamber at the current time step. The first term on the right hand side represents the mass of air at the previous time step. The second term on the right hand side represents the amount of mass entering into the chamber during the interval considered. It may be noted here that an implicit method requires flowrate at the current time step.

The mass flowrate, $\dot{m}$, in equation 1 is calculated from the Bernoulli equation that expresses the relationship between the stagnation pressure of the supply line and the static pressure in the tank.

$$\dot{m} = \sqrt{\frac{2\gamma}{(\gamma - 1)} p_s \rho_s \left(\frac{p}{p_s}\right)^{2/\gamma} \left(1 - \frac{p}{p_s}\right)^{\gamma-1/\gamma}} \qquad (2)$$

2.2 Energy Conservation Equation :

The energy conservation equation can be written as

$$(mu)_{t+\Delta t} = (mu)_t + \dot{m}_{t+\Delta t}\, h_s \qquad (3)$$

The expression for the internal energy at the current time step consists of two terms. The first term on the right hand side of equation (3) represents the energy at the previous time step. The amount of energy entering the control volume, during the current time step, is represented by the second term in the equation. Temperature at the current time step is calculatedfrom the internal energy, $u_{t+\Delta t}$

$$T_{t+\Delta t} = \frac{u_{t+\Delta t}}{C_v} \qquad (4)$$

2.3 Equation of State:

During transient pressurization, it is assumed that thermodynamic equilibrium exists. Therefore, the equation of state for an ideal gas is used to calculate the pressure in the chamber.

$$p_{t+\Delta t} = \frac{m_{t+\Delta t}\, R\, T_{t+\Delta t}}{V} \qquad (5)$$

The need for a numerical solution stems from the fact that equation (2) is non-linear and equation (2) and (5) are coupled. It may further be noted here that for the isothermal model equation (3) is not required to be solved. Temperature is constant and equals the supply temperature during the entire pressurization process.

2.4 Solution Procedure:

An implicit successive substitution method is used to solve the governing equations(Eqns. 1 through 5). Figure 2 shows the information flow diagram. A FORTRAN program has been constructed and the solution steps, illustrated in Figure 3, are as follows:

1. Initially (at t = 0);
   p = 0,
   T = 0, and
   m = 0.

2. Begin a new timestep ($t = t + \Delta t$) and update variables in the current timestep.

3. Begin the iteration loop for the current timestep and make a guess of the variable values. Usually in unsteady computation, the values prevailing at the previous timestep are the best guess.

4. Calculate resident mass, $m_{t+\Delta t}$, from the mass conservation equation (Eqn. 1)

5. Calculate internal energy from the energy conservation equation (Eqn. 3)

6. Calculate the pressure in the chamber from the equation of state (Eqn. 5)

7. Calculate the fractional difference in pressure between successive iterations.

8. If the fractional difference is larger than a pre-specified small number known as the convergence criterion (convergence criterion for the present problem is set to 1.E-06), the iteration loop is repeated (steps 3 to 7) until the solution is converged.

3.0 Results & Discussion

To develop a generalized solution of the pressurization process, the physical parameters that can influence the process must be identified. For a given fluid, chamber volume, vent area, supply pressure and temperature can influence the pressurization processes. To study the effect of these four parameters, 9 test cases are selected and are described in Table 1. In order to prepare generalized charts appropriate dimensionless numbers are required to be defined. The dimensionless numbers are defined by dividing the physical quantities such as pressure, mass and mass flowrate by respective characteristic values. Following dimensionless numbers are identified for a pressurization process.

Non-dimensional pressure, $\hat{p} = p/p_s$

Non-dimensional mass, $\hat{m} = m/m_{fill}$

Non-dimesional flowrate, $\hat{m}' = m'/m'_{choked}$

Non-dimensional time, $\hat{t} = t\, d^2\, (\gamma R T_s)^{1/2} / V$

It is relatively straightforward to define the characteristic values of pressure, mass and massflowrate. The characteristic pressure is the supply pressure for the pressurization case. The characteristic mass is the mass of air at the end of the pressurization process. The characteristic mass flowrate is the choked mass flowrate that occurs in the beginning of the process.. The identification of characteristic time is a little more involved. It has been defined as:

Characteristic Time = $V/ d^2 (\gamma R T_s)^{1/2}$

It is a ratio of a characteristic distance and a characteristic velocity. The characteristic distance is defined to be the ratio of volume and area and the characteristic velocity is the speed of sound at the supply condition.

3.1 Effect of geometry

The effect of geometry on predicted pressure history during isentropic pressurization is shown in Figure 4. Two geometric parameters are considered: chamber volume and vent diameter. Cases 1, 2 and 3 show the effect of volume. The chamber volume was 1 cu. m and vent diameter is 1 cm for case 1. The chamber volume was reduced to 0.5 cu. m in case 2 and increased to 2 cu.m in case 3 while the vent diameter remained the same in cases 1,2 and 3. The time required to pressurize the chamber to the supply value of about 1.4E6 N/m**2(approx. 200 psia) is about 40 sec. When the volume was reduced to 50% of its original size(case 2), the time required for pressurization was also reduced in the same proportion. Increasing the volume to 2 cu.m doubles the pressurization time to about 80 seconds in case 3. This set of parametric studies show that the pressurization time is directly proportional to chamber volume. Cases 1, 4 and 5 are also plotted in Figure 4 to show the effect of diameter. The vent diameter was reduced by half in case 4 and doubled in case 5. The time required for pressurization was 160 seconds and 10 seconds respectively. This case study shows that the pressurization time is inversely proportional to the square of the vent diameter.

The effect of geometry on predicted mass history during isentropic pressurization is shown in Figure 5. The effect of chamber volume on mass history can be observed from the results of case 1, 2 and 3. It is observed that the predicted mass at the end of the pressurization is directly proportional to the chamber volume. However the rate of increase in mass depends on the diameter of the vent. For example, cases 1, 4 and 5 have the same volume but different vent diameters. In all three cases the chamber receives the same mass of air. The required time was different in all three cases because of the change in vent diameter.

The effect of geometry on predicted mass flow rate history during isentropic pressurization is

shown in Figure 6. During the initial period of pressurization the vent is choked and the mass flowrate remains constant. As the pressure in the chamber increases, the flow becomes subsonic and gradually decreases to zero. The choked flowrate is proportional to area of the vent.

3.2 Effect of supply pressure

The effect of supply pressure (cases 1, 6 and 7) on isentropic pressurization is shown in Figures 7, 8 and 9 . The predicted pressure history for three cases is shown in Figure 7. It is interesting to observe that pressurization time is independent of supply pressure. It may first appear that higher supply pressure should reduce the pressurization time. One may tend to make a such conclusion by observing the results in Figure 9. Figure 9 shows the predicted mass flowrate history for these three cases. The mass flowrate is higher for higher pressure. It must also be recognized that higher pressure requires a larger amount of air mass to pressurize the chamber to the supply value. Since two opposing effects cancel each other, the pressurization time remains unchanged with pressure variation. This is evident from figure 8 which shows the predicted mass history for three different supply pressures. It is observed that pressurization time for all three cases was 40 seconds. However the mass of air supplied to the chamber in cases 1, 6 and 7 are 6.07 kg, 4.34 kg and 8.68 kg respectively.

3.3 Effect of supply temperature

The influences of supply temperature (cases 1, 8 and 9) on pressure history, mass history and mass flowrate history during isentropic pressurization are shown in Figures 10, 11 and 12. The pressurization time is found to increase (Figure 10) with the reduction in supply temperature. Since air has a higher density at lower temperature, the amount of mass supplied to the chamber is higher at lower temperature as evident from Figure 11. For same reason case 8 predicts larger mass flowrate than cases 1 and 9 as shown in Figure 12.

3.4 Generalized curves for isentropic pressurization

The dimensionless pressure, mass and mass flowrate history are plotted as a function of dimensionless time as shown in Figures 13, 14 and 15 respectively. It is evident that all 9 cases collapse into a single curve in these three figures. These figures can be used to determine the pressure, mass and mass flowrate history of any arbitrary geometry and supply condition during an isentropic pressurization process.

3.5 Generalized curves for isothermal pressurization

Parametric calculations described in Table 1 have been repeated for isothermal condition. For the purpose of brevity, the results of the parametric study have been omitted from the paper. However, dimesionless pressure, mass and mass flowrate are plotted in figures 16, 17 and 18 respectively. The isentropic results are also plotted for the purpose of comparison. It may be observed that isothermal pressurization takes longer and fills up the chamber with a larger

mass of air. Both observations agree with each other considering the fact that the maximum flowrate is same for two cases.

3.6 Pressurization Number

Non-dimensional time, pressure, mass and mass flowrate for isentropic and isothermal pressurization have been provided in Tables 2 and 3. For any given chamber volume, vent size and supply temperature, pressure, mass and mass flowrate at any time during pressurization can be estimated from Tables 2 and 3. A new dimensionless number to characterize pressurization has been identified in the present study. The dimensionless group can be expressed as :

$$P = \frac{t_{fill} d^2 \sqrt{\gamma R T_s}}{V}$$

The predicted pressurization number for isentropic and isothermal cases are 1.9465 and 2.7239 respectively. For any given chamber volume, vent size and supply temperature fill up time can be calculated from this number.

4. Conclusions

An implicit successive substitution method has been developed to model a pressurization process in a chamber. Parametric computations have been performed to study the effect of chamber volume, vent size, supply pressure and temperature. From parametric studies a new dimensionless group has been identified to characterize the pressurization process. Tables and charts have been developed to predict the time history of pressure, mass and mass flowrate for isentropic and isothermal process.

5. References

1. F. J. Moody, Introduction to Unsteady Thermofluid Mechanics , John Wiley & Sons, Inc., 1990.

2. G. J. Van Wylen and R. E. Sonntag , Fundamentals of Classical Thermodynamics , John Wiley & Sons, 1976.

6. Acknowledgement

The work has been performed under task number 651-006 of NASA/MSFC Contract No. NAS 8-37814. Mr. Robert Fisher is the Task Manager of the aforementioned task. The author also acknowledges the contribution of Lawrence Chesser and S. Bukkapatnam of Sverdrup Technology.

7. Nomenclature

A	Area of vent
C_v	Specific heat of air at constant volume
d	Diameter of vent
h	enthalpy
m	mass of air
$\dot{m}$	mass flow rate
p	pressure
R	Gas constant
T	temperature
t	time
u	internal energy
V	volume
ρ	density
γ	specific heat ratio

subscript

s	supply condition

Table 1. Test cases for parametric study

Case No.	P (Pa)	T (R)	V (m^3)	d (cm)
1.	1.4E6	573.15	1.00	1.00
2.	1.4E6	573.15	0.50	1.00
3.	1.4E6	573.15	2.00	1.00
4.	1.4E6	573.15	1.00	0.50
5.	1.4E6	573.15	1.00	2.00
6.	1.0E6	573.15	1.00	1.00
7.	2.0E6	573.15	1.00	1.00
8.	1.4E6	473.15	1.00	1.00
9.	1.4E6	673.15	1.00	1.00

Table 2. Pressurization Table for the Isothermal Case

$\bar{t}$	$\bar{p}$	$\bar{m}$	$\bar{m}'$
.9598E-01	.6107E-01	.6107E-01	.1000E+01
.1920E+00	.1221E+00	.1221E+00	.1000E+01
.2879E+00	.1832E+00	.1832E+00	.1000E+01
.3839E+00	.2443E+00	.2443E+00	.1000E+01
.4799E+00	.3054E+00	.3054E+00	.1000E+01
.5759E+00	.3664E+00	.3664E+00	.1000E+01
.6718E+00	.4275E+00	.4275E+00	.1000E+01
.7678E+00	.4886E+00	.4886E+00	.1000E+01
.8638E+00	.5496E+00	.5496E+00	.9990E+00
.9598E+00	.6103E+00	.6103E+00	.9850E+00
.1056E+01	.6696E+00	.6696E+00	.9547E+00
.1152E+01	.7266E+00	.7266E+00	.9084E+00
.1248E+01	.7802E+00	.7802E+00	.8467E+00
.1344E+01	.8297E+00	.8297E+00	.7708E+00
.1440E+01	.8740E+00	.8740E+00	.6818E+00
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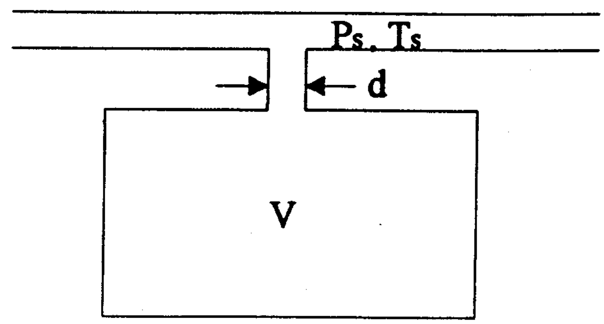
.1632E+01	.9448E+00	.9448E+00	.4708E+00
.1728E+01	.9699E+00	.9699E+00	.3525E+00
.1824E+01	.9876E+00	.9876E+00	.2282E+00
.1920E+01	.9976E+00	.9976E+00	.1004E+00
.1946E+01	.9990E+00	.9990E+00	.6430E-01

Table 3. Pressurization Table for the Isothermal Case

$\bar{t}$	$\bar{p}$	$\dot{\bar{m}}$	$\dot{\bar{m}}'$
9.60E-02	4.36E-02	6.11E-02	1.00E+00
1.92E-01	8.73E-02	1.22E-01	1.00E+00
2.88E-01	1.31E-01	1.83E-01	1.00E+00
3.84E-01	1.75E-01	2.44E-01	1.00E+00
4.80E-01	2.18E-01	3.05E-01	1.00E+00
5.76E-01	2.62E-01	3.66E-01	1.00E+00
6.72E-01	3.05E-01	4.28E-01	1.00E+00
7.68E-01	3.49E-01	4.89E-01	1.00E+00
8.64E-01	3.93E-01	5.50E-01	1.00E+00
9.60E-01	4.36E-01	6.11E-01	1.00E+00
1.06E+00	4.80E-01	6.72E-01	1.00E+00
1.15E+00	5.24E-01	7.33E-01	1.00E+00
1.25E+00	5.67E-01	7.94E-01	9.97E-01
1.34E+00	6.10E-01	8.54E-01	9.85E-01
1.44E+00	6.53E-01	9.14E-01	9.65E-01
1.54E+00	6.94E-01	9.72E-01	9.37E-01
1.63E+00	7.35E-01	1.03E+00	9.01E-01
1.73E+00	7.73E-01	1.08E+00	8.56E-01
1.82E+00	8.09E-01	1.13E+00	8.05E-01
1.92E+00	8.43E-01	1.18E+00	7.47E-01
2.02E+00	8.74E-01	1.22E+00	6.82E-01
2.11E+00	9.02E-01	1.26E+00	6.11E-01
2.21E+00	9.27E-01	1.30E+00	5.35E-01
2.30E+00	9.49E-01	1.33E+00	4.54E-01
2.40E+00	9.67E-01	1.35E+00	3.70E-01
2.50E+00	9.81E-01	1.37E+00	2.82E-01
2.59E+00	9.91E-01	1.39E+00	1.92E-01
2.69E+00	9.98E-01	1.40E+00	9.99E-02
2.72E+00	9.99E-01	1.40E+00	6.49E-02



Given : Supply Pressure, Ps
Supply Temperature, Ts
Passage Diameter, d
Bottle Volume, V

Calculate: Pressure History, P(t)
Mass History, m(t)
Mass Flowrate History, m'(t)
Fill-up Time

Figure 1. Schematic of the Pressurization process

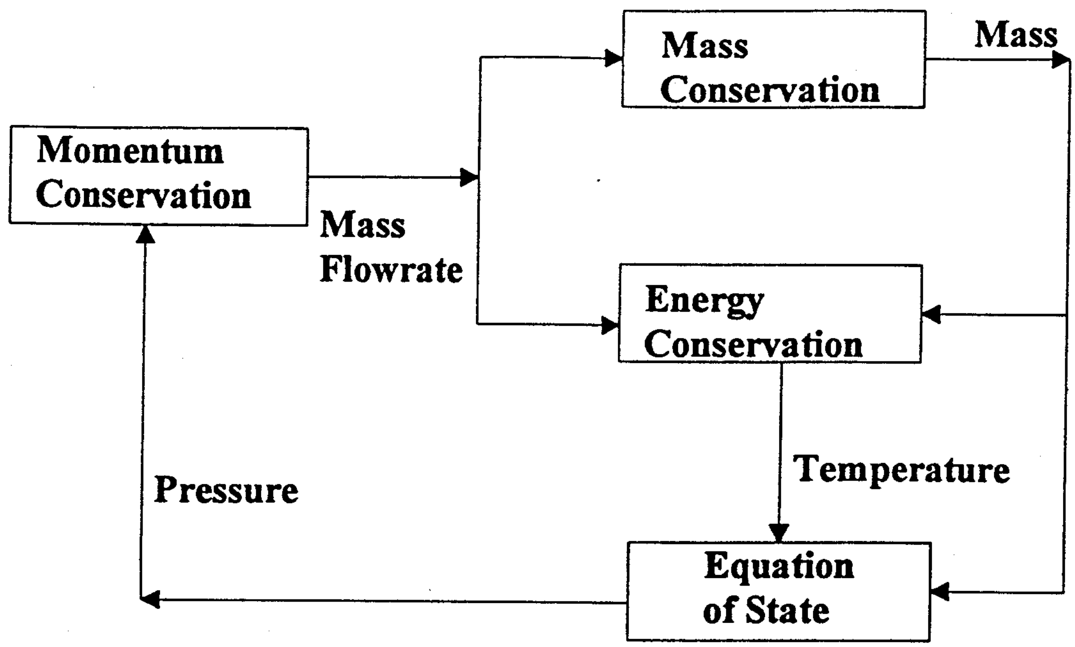


Figure 2. Information flow diagram of the successive substitution algorithm

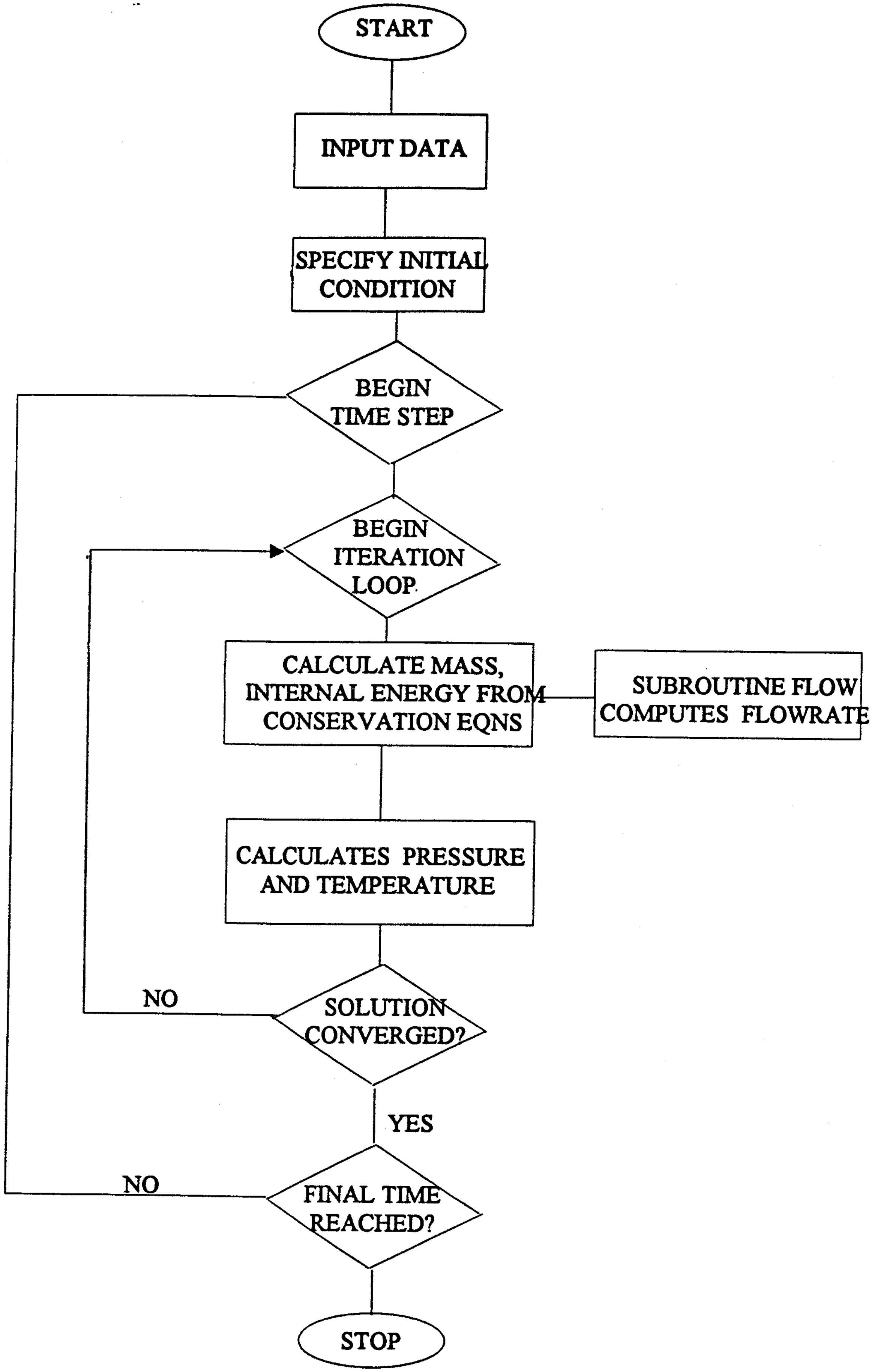


Figure 3. Flow Diagram of the Computer Program, FLAP

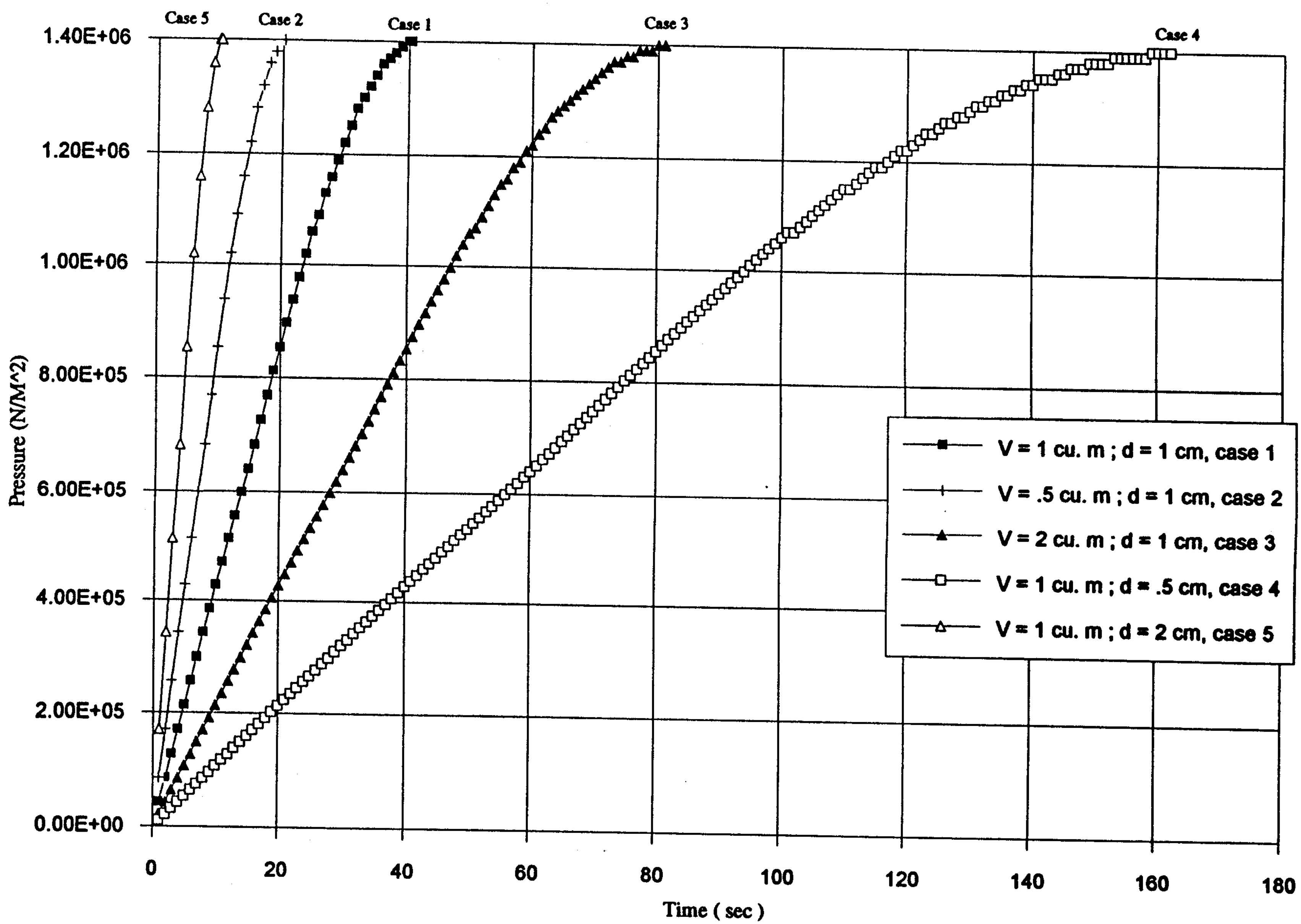


Fig 4: Effect of geometry on predicted pressure history during tank pressurization

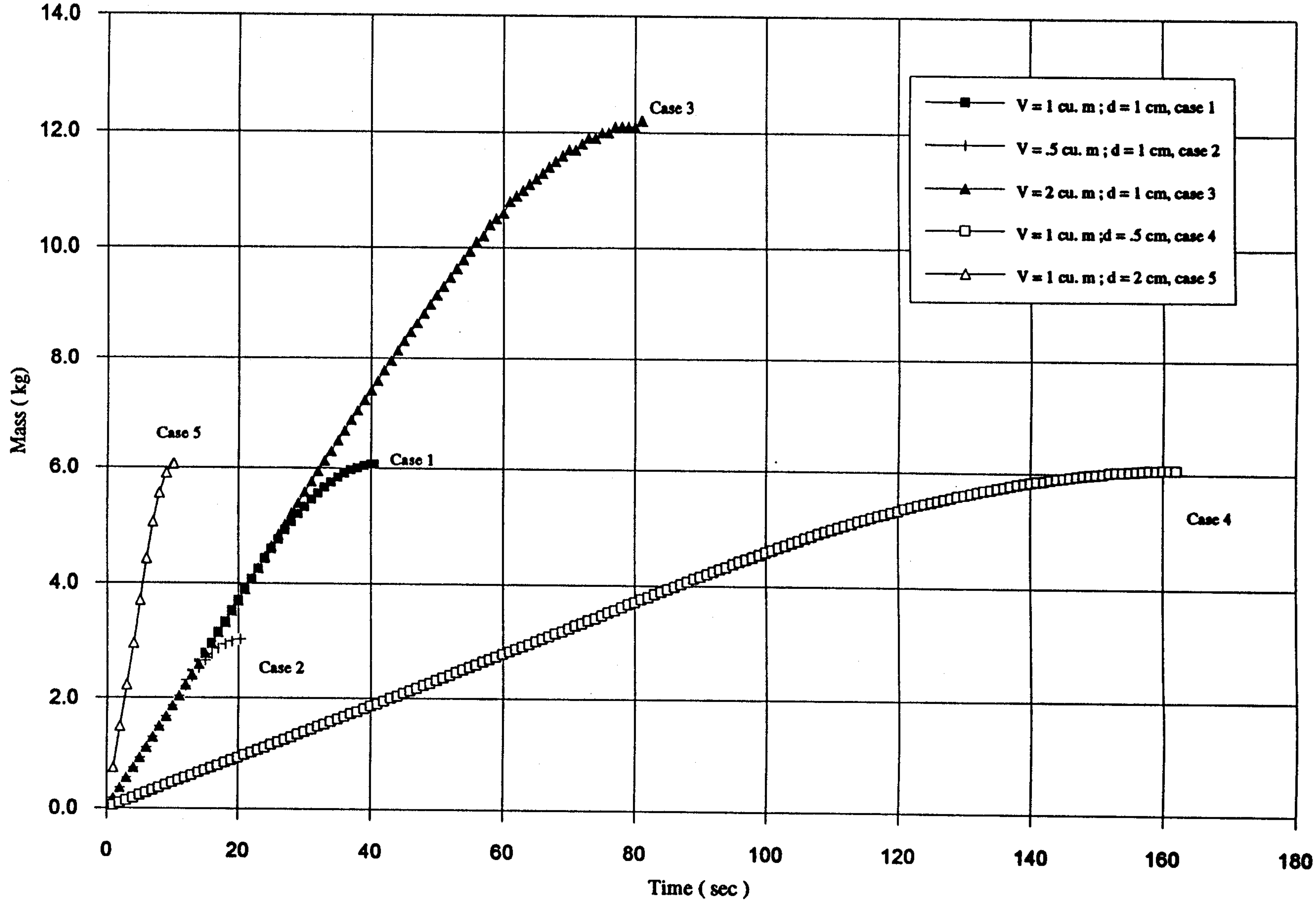


Fig 5: Effect of geometry on predicted mass history during tank pressurization

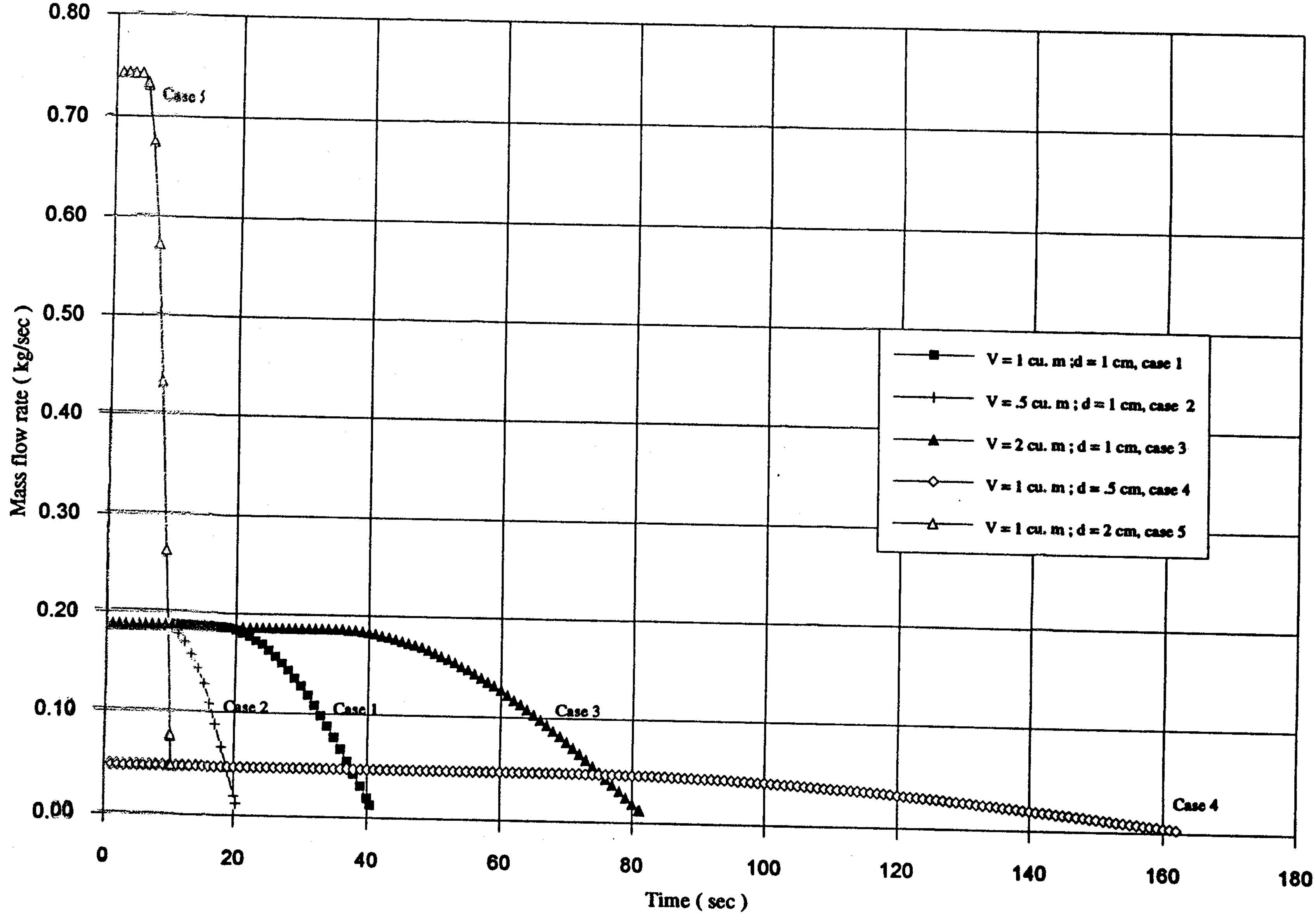


Fig 6: Effect of geometry on predicted mass flow rate during tank pressurization

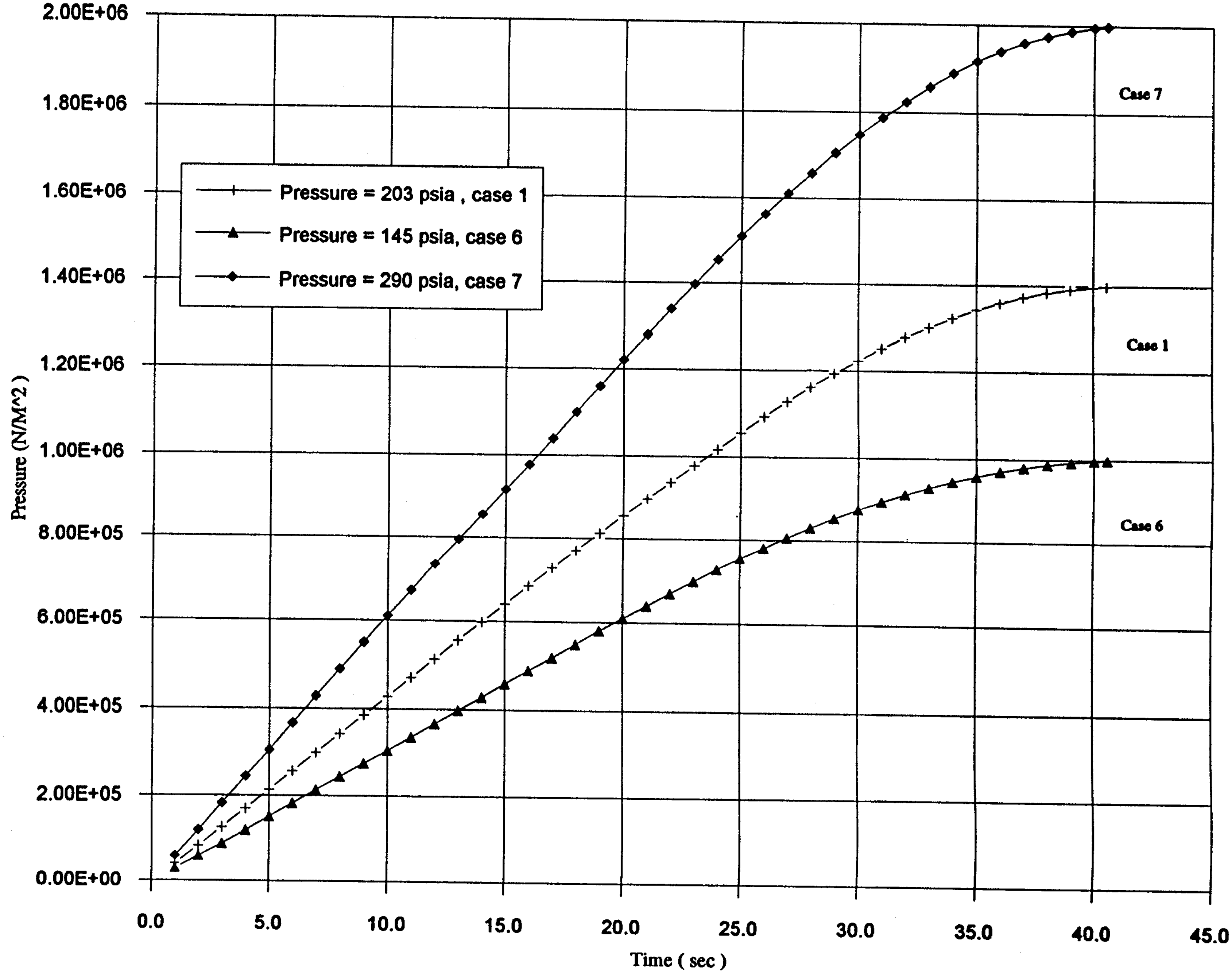


Fig 7: Effect of supply pressure on predicted pressure history during tank pressurization

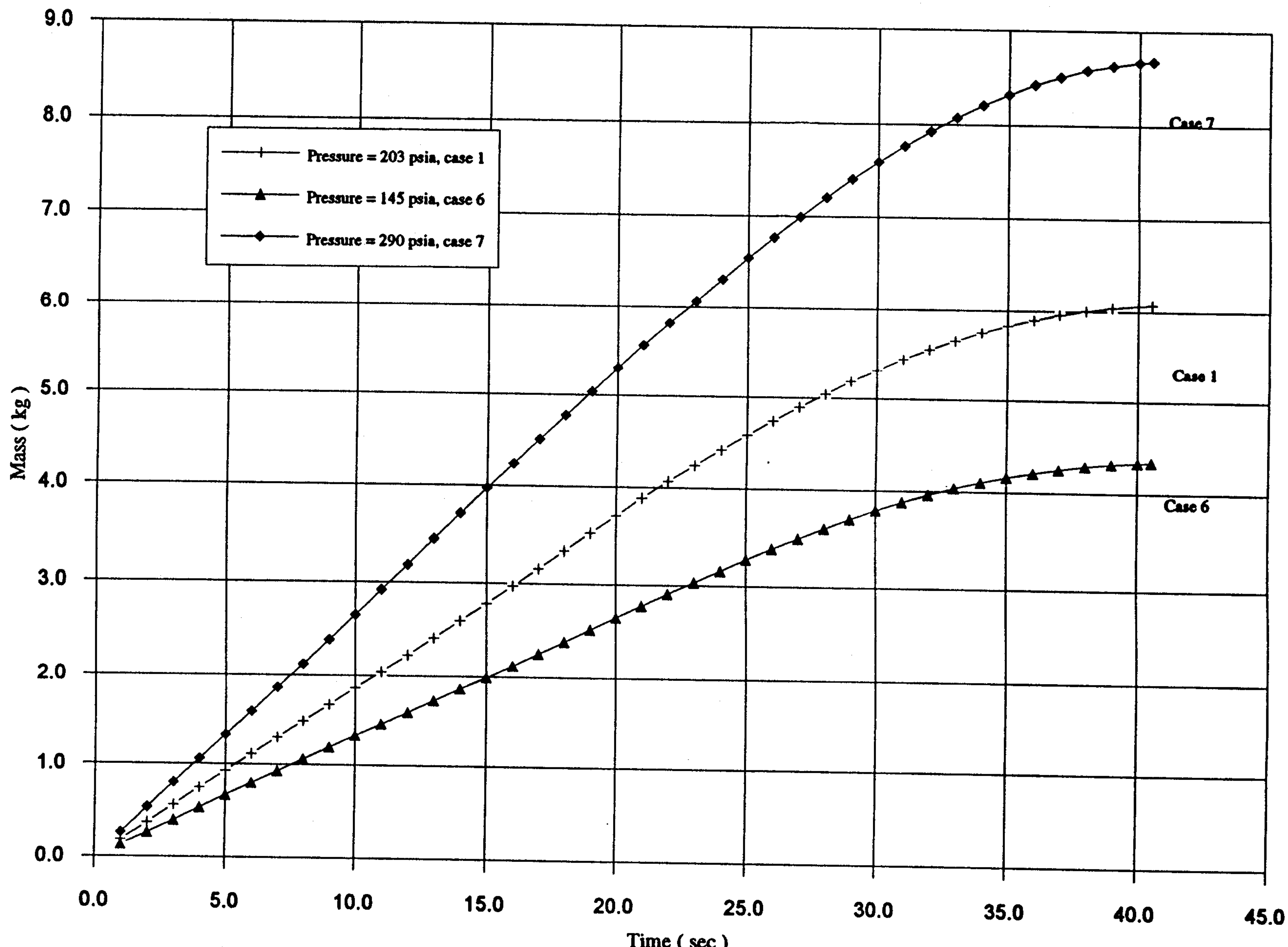


Fig 8: Effect of supply pressure on predicted mass history during tank pressurization

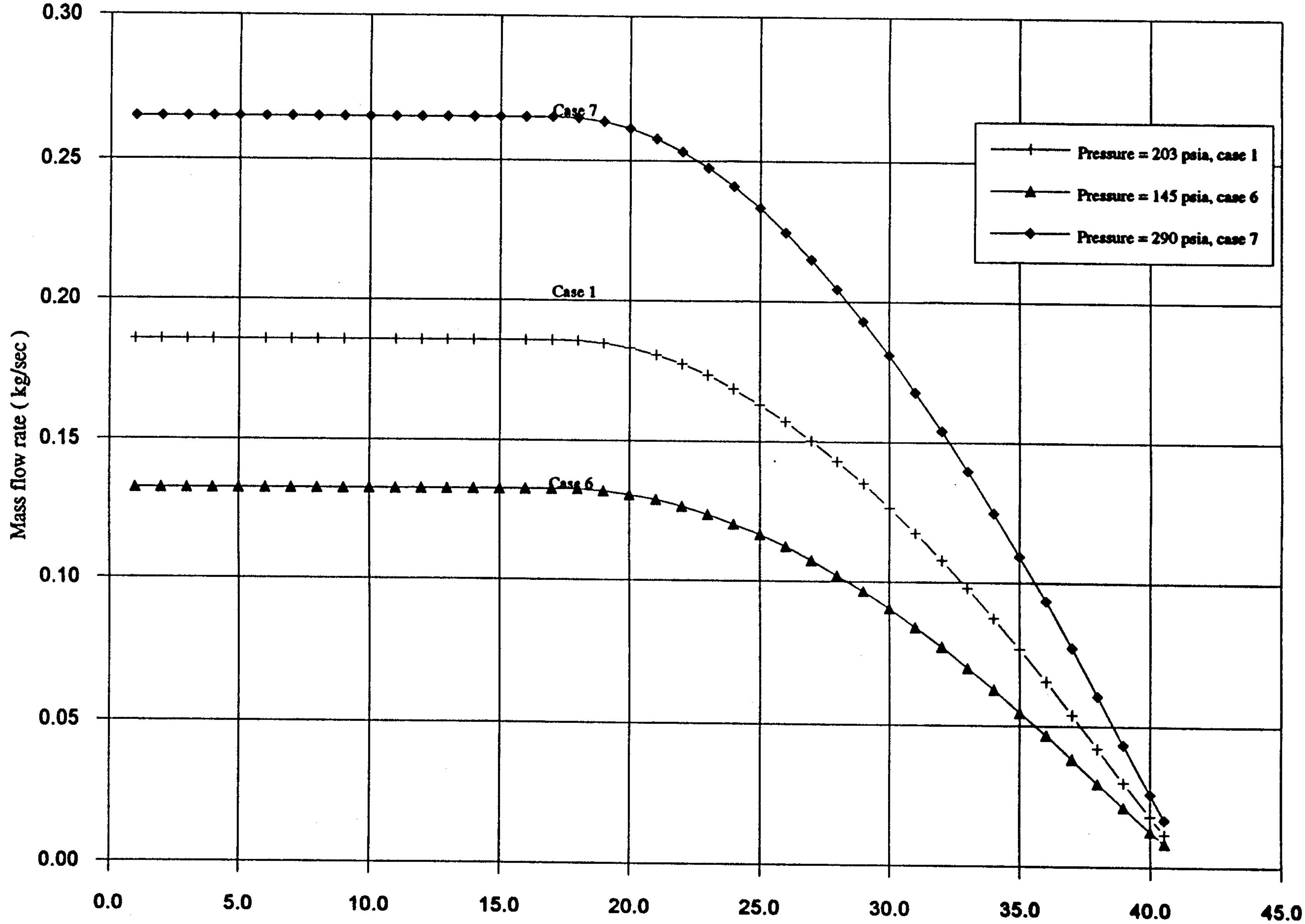


Fig 9: Effect of supply pressure on predicted mass flow rate history during tank pressurization

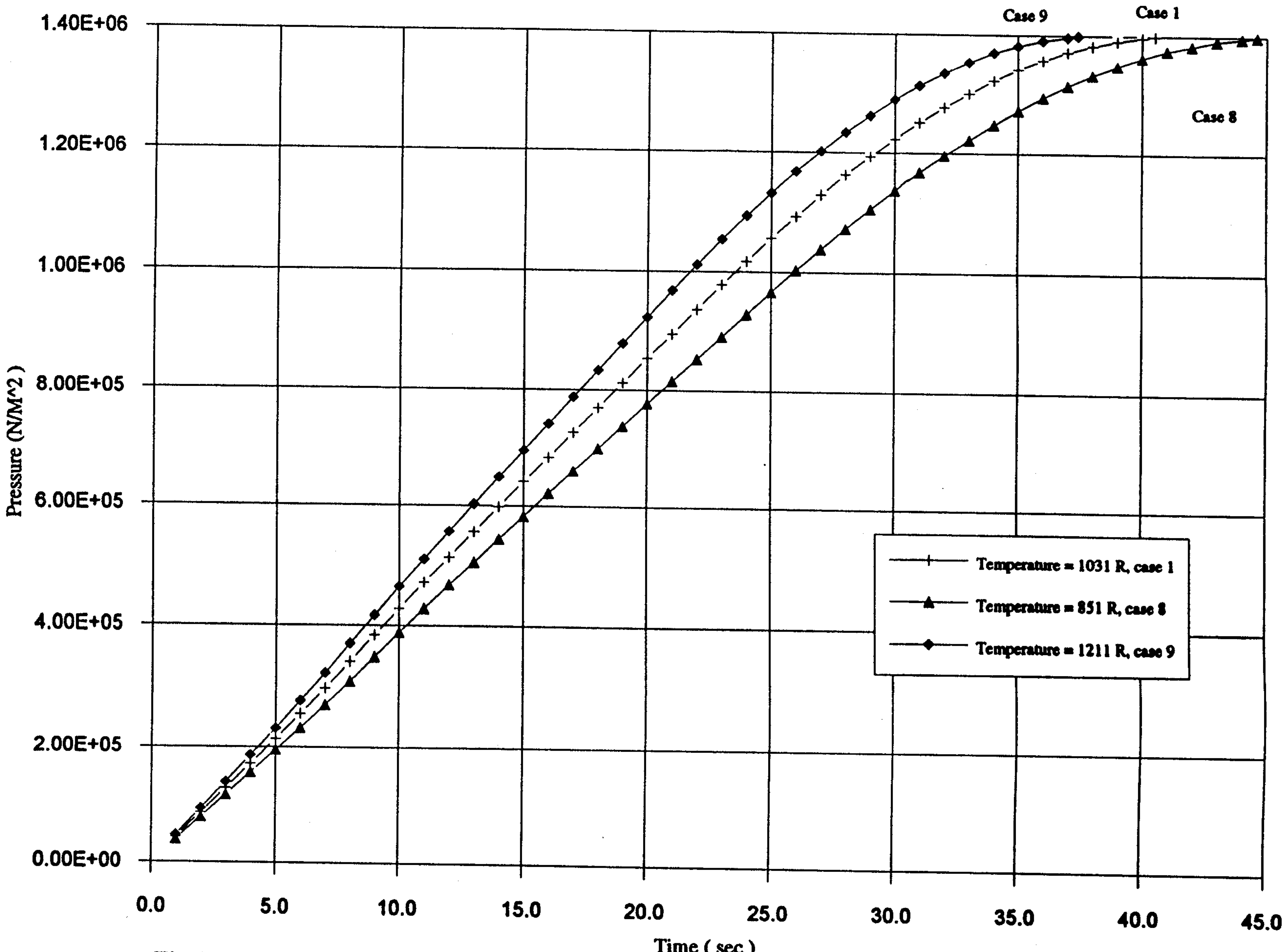


Fig 10: Effect of supply temperature on predicted pressure history during tank pressurization

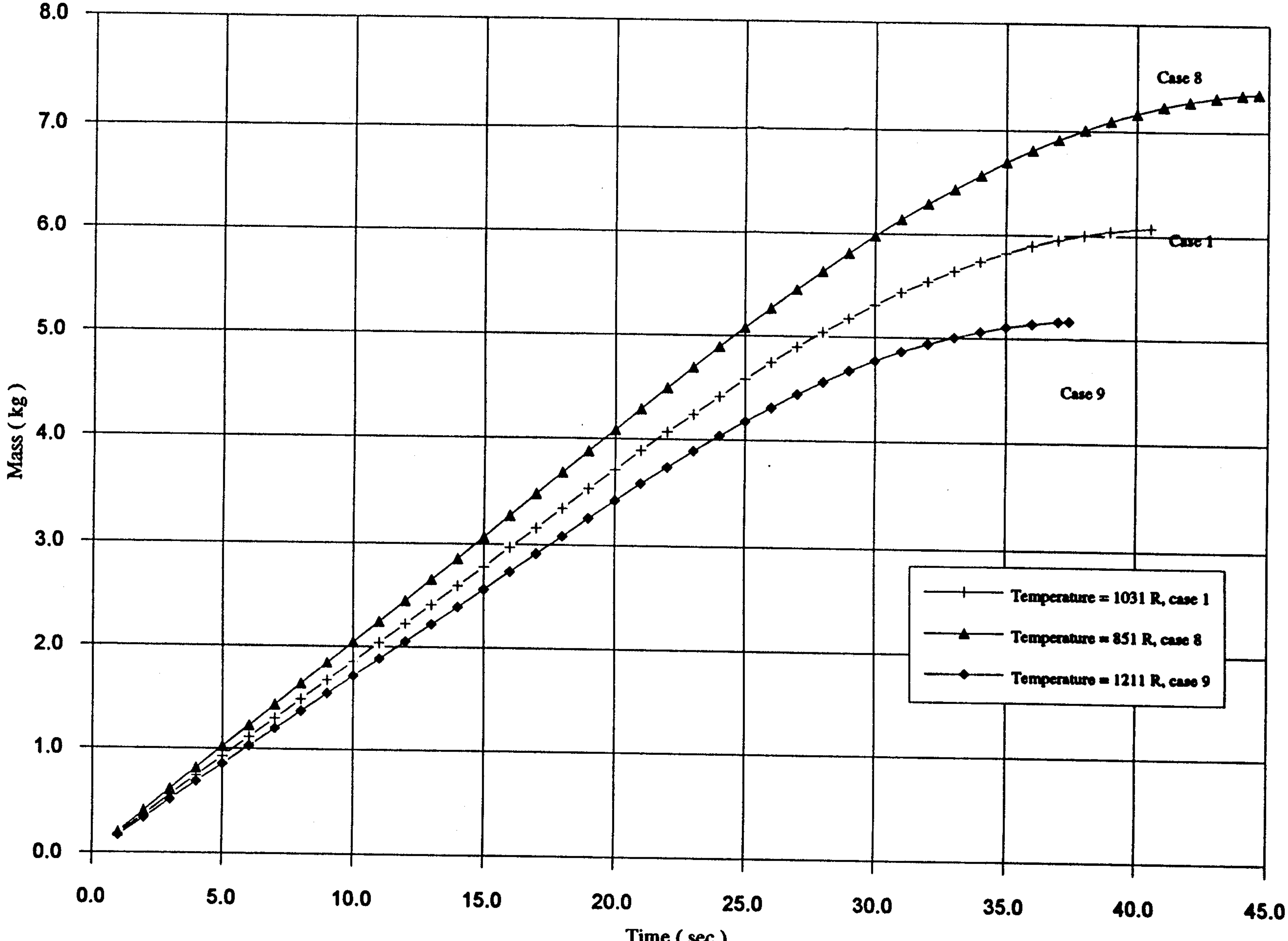


Fig 11: Effect of supply temperature on mass history during tank pressurization

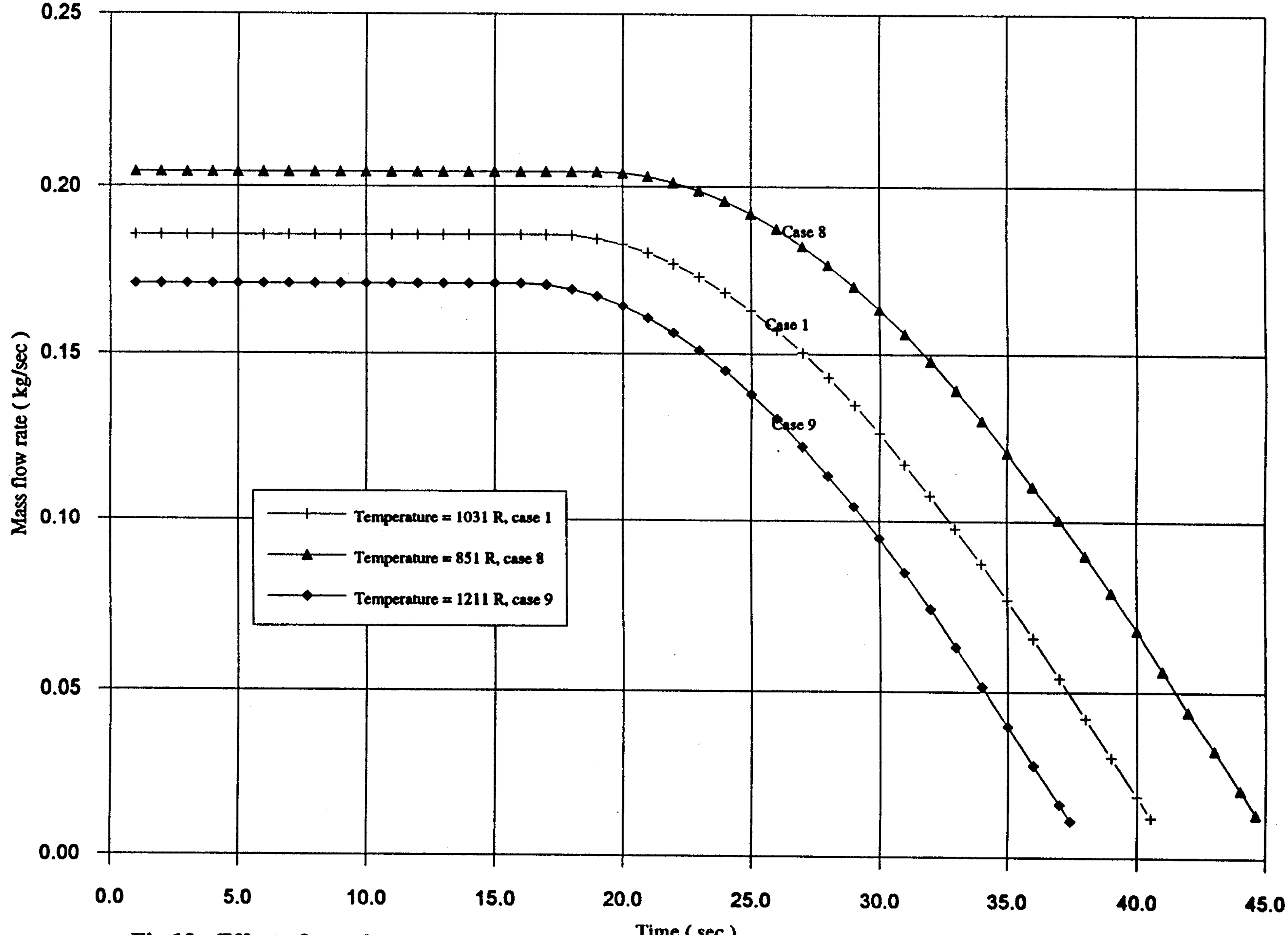


Fig 12: Effect of supply temperature on predicted mass flow rate history during tank pressurization

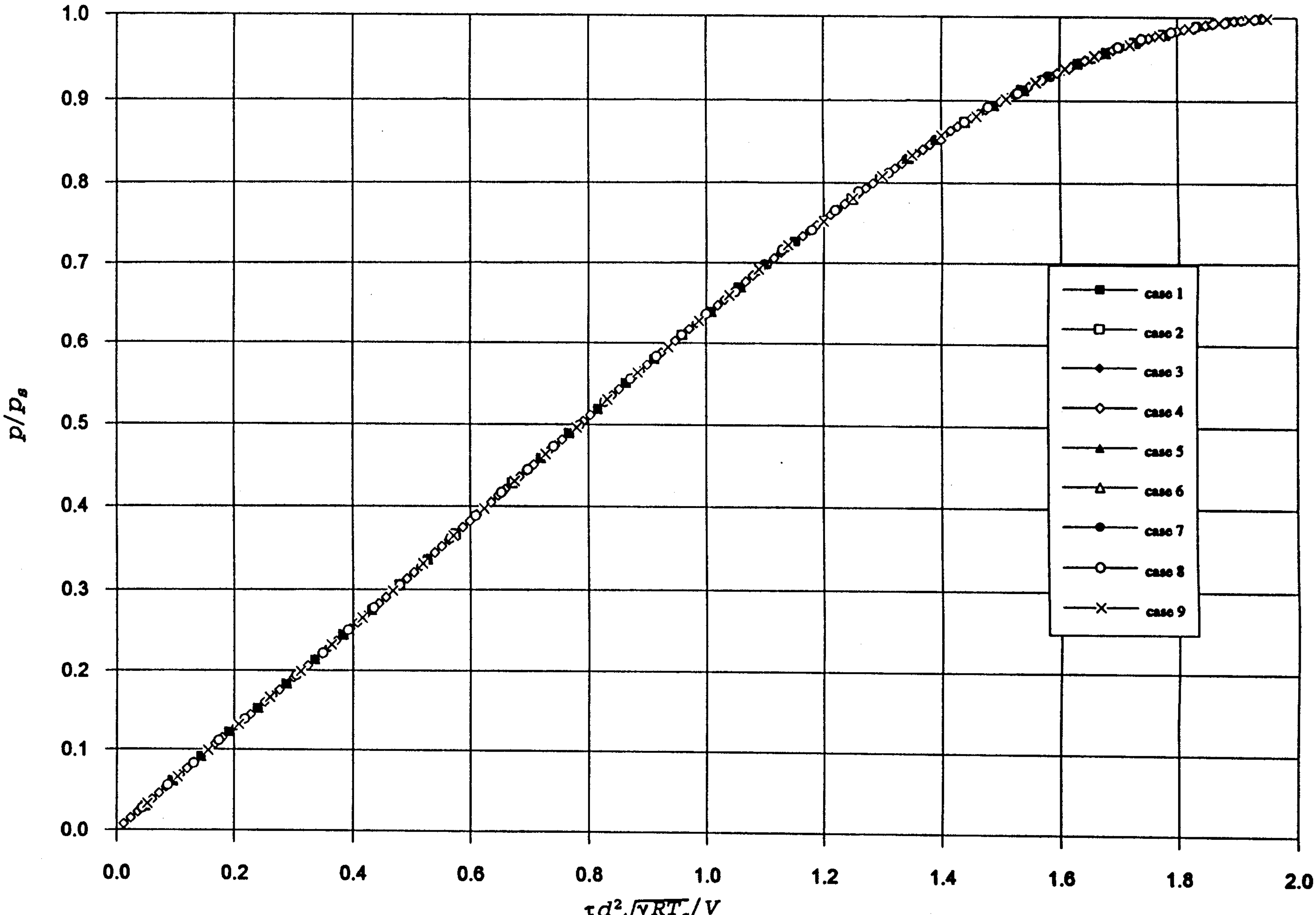


Fig 13: Dimensionless pressure history during isentropic pressurization

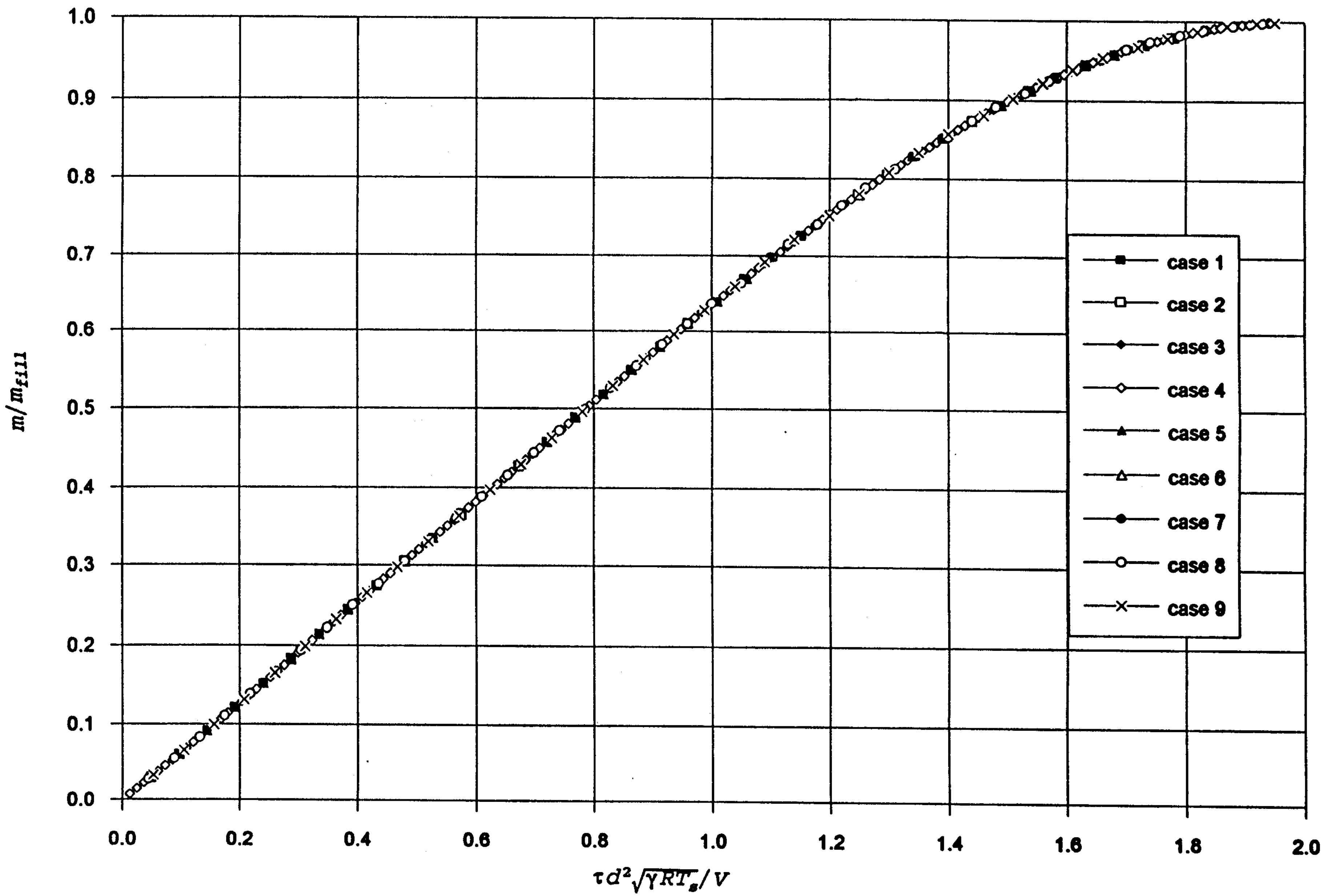


Fig 14: Dimensionless mass history during isentropic pressurization

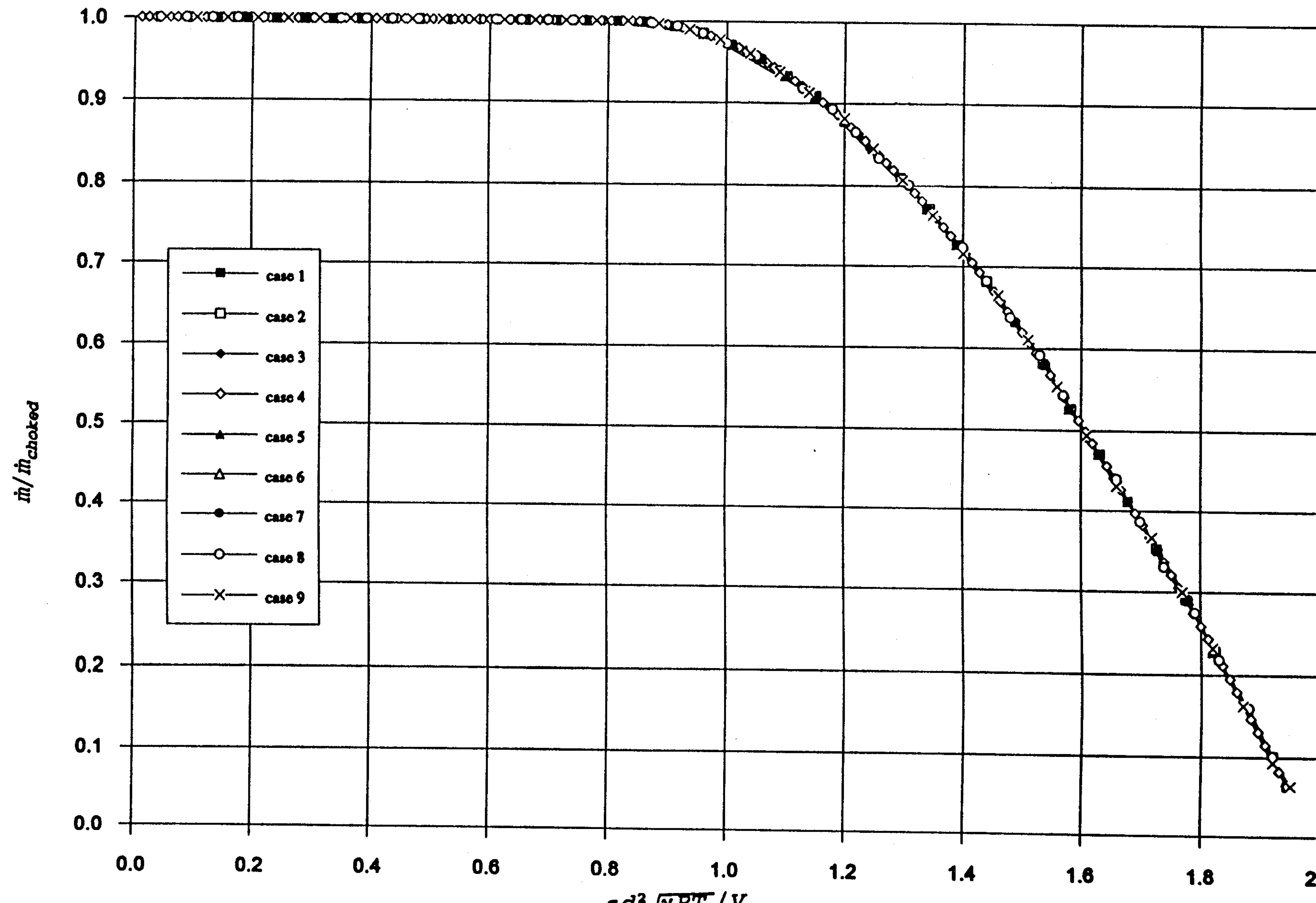


Fig 15: Dimensionless mass flow rate history during isentropic pressurization

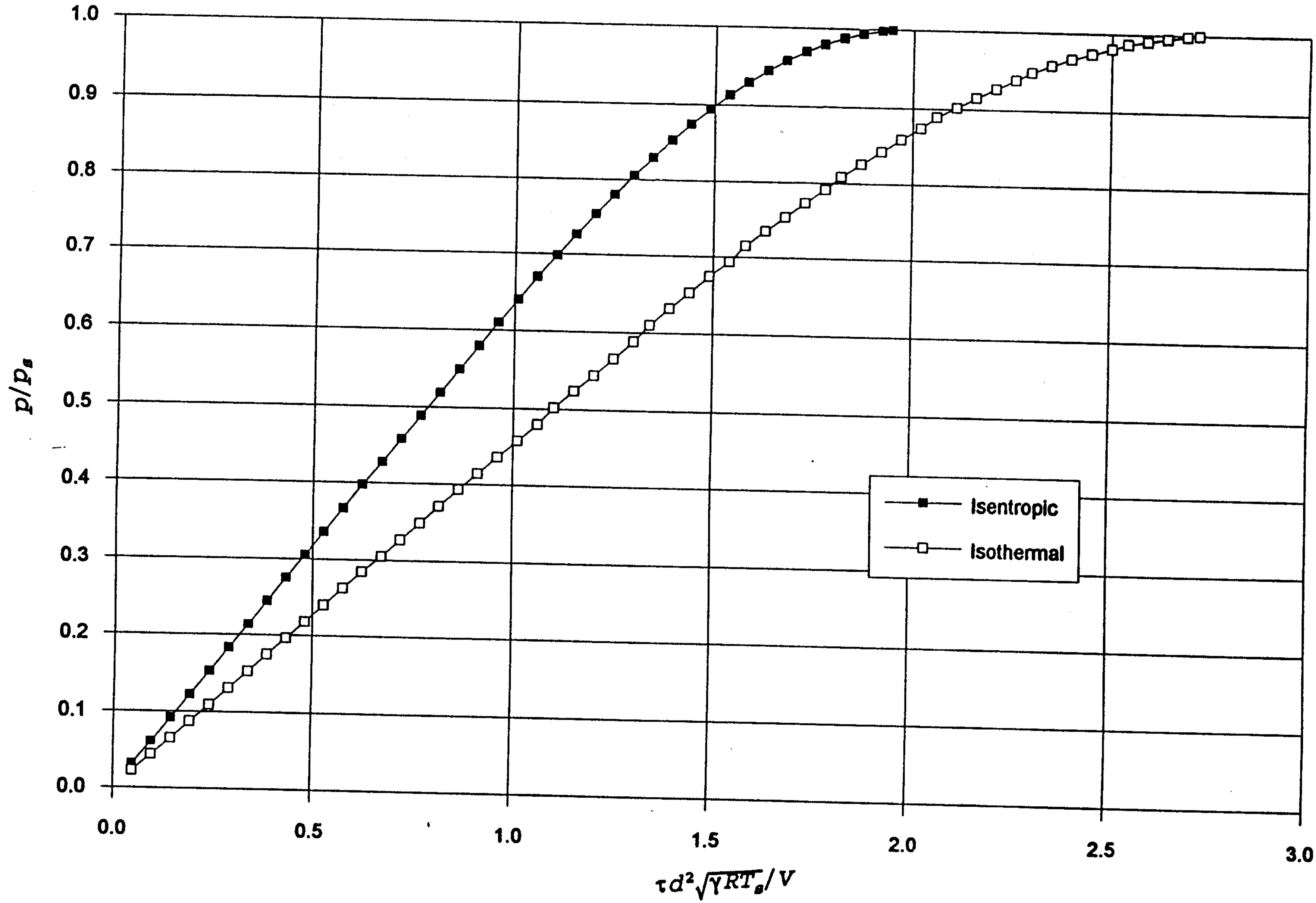


Fig 16: Dimensionless pressure history during isentropic and isothermal pressurization

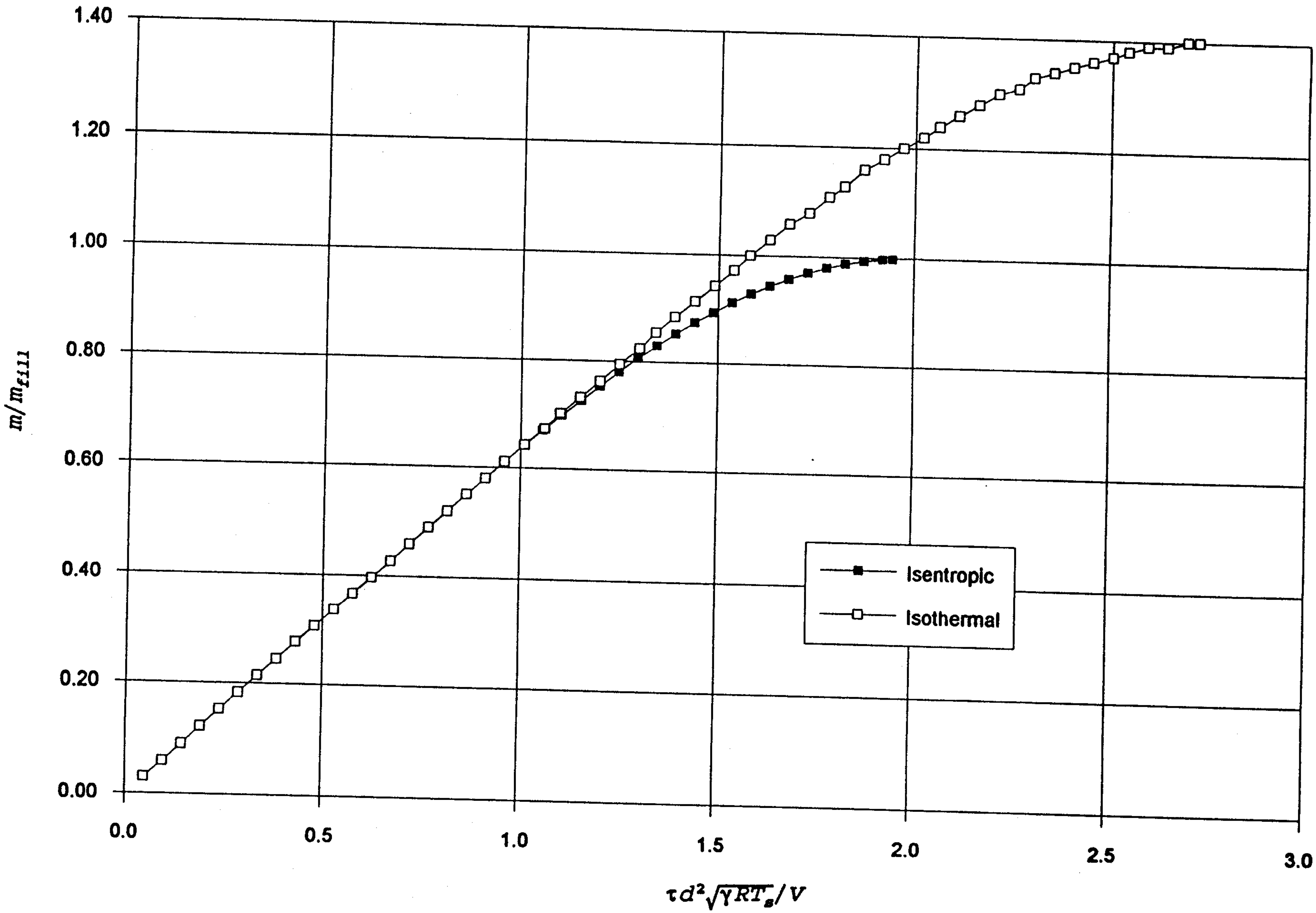


Fig 17: Dimensionless mass history during isentropic and isothermal pressurization